

The Isochoric Heat Capacity of Ice is Equal to its Isobaric Heat Capacity

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Received : October 11, 2023

Published : October 26, 2023

ABSTRACT

Formerly the author theoretically obtained the result that the isochoric heat capacity is equal to the isobaric heat capacity. In the present paper this conclusion is compared with experimental results for H₂O ice Ih. A sound agreement between the theory and experiment is observed.

Keywords: Reech's relation; Ice; generalized Mayer's relation; Heat capacity; isothermal compressibility; adiabatic compressibility; the First law of thermodynamics

INTRODUCTION

In [1] the author developed a theory that the isochoric heat capacity C_V is equal to the isobaric heat capacity C_P . It is necessary to confirm that conclusion with experimental results. Relevant experimental results for H₂O ice Ih were published in [2] and using them one can check the theoretical results of [1].

THEORY

The first law of thermodynamics for the heat exchange can be written like this in its traditional form:

$$dQ = dU + PdV. \quad (1)$$

Here dQ is the heat introduced into the system. It is an exact differential because pressure is constant.

In [1] it was shown that this law must have the following form

$$dQ = dU + PdV + VdP. \quad (2)$$

Pay attention that dQ is an exact differential. For a constant pressure, Eq. 2 turns to Eq. 1, and for a constant volume, Eq. 2 turns to the following equation:

$$\delta Q = dU + VdP. \quad (3)$$

The traditional heat capacities C_P and C_V are obtained from Eq. 1.

There is Reech's relation [3,4]:

$$\gamma = \frac{C_P}{C_V} = \frac{K_T}{K_S}. \quad (4)$$

Here K_T and K_S are isothermal and adiabatic compressibility, respectively. In the derivation of Reech's relation only

reciprocity theorem is used [4] and this derivation is independent of a form of the first law of thermodynamics: Eqs. 1 or 2.

In [2] numerous values of K_T and K_S for water Ice Ih were presented: Table 1.

Table 1: Isothermal compressibility K_T , and adiabatic compressibility K_S , for ice Ih [2]^a

T, K	$K_T, 10^{-10} \text{Pa}^{-1}$	$K_S, 10^{-10} \text{Pa}^{-1}$	γ , Eq. 4
0	0.8825	0.8825	1,0000
10	0.8880	0.8880	1.0000
20	0.8936	0.8935	1.0001
30	0.8992	0.8990	1.0002
40	0.9049	0.9047	1.0002
50	0.9094	0.9093	1.0001
60	0.9160	0.9160	1.0000
70	0.9229	0.9228	1.0001
80	0.9304	0.9301	1.0003
90	0.9384	0.9376	1.0009
100	0.9471	0.9456	1.0016
110	0.956	0.9539	1.0022
120	0.9660	0.9626	1.0035
130	0.9768	0.9717	1.0053
140	0.9880	0.9813	1.0068
150	0.9997	0.9913	1.0085
160	1.012	1.002	1.0100
170	1.026	1.013	1.0128
180	1.039	1.024	1.0147
190	1.054	1.036	1.0174
200	1.069	1.049	1.0191
210	1.085	1.062	1.0217
220	1.102	1.076	1.0242
230	1.119	1.091	1.0257
240	1.137	1.106	1.0280
250	1.155	1.122	1.0294
260	1.139	1.173	0.9710
270	1.191	1.157	1.0294
273	1.196	1.162	1,0293

^aThe uncertainties for K_S and K_T are $\pm 1.3\%$, and $\pm 1.5\%$, respectively.

DISCUSSIONS AND CONCLUSIONS

The data presented in Table 1 provide good evidence that Eq. 2 formerly obtained by the author is correct. However, further research is needed to prove that finally. These data

confirm well the result obtained by A. Guy: $|P\Delta V| = |V\Delta P|$ [1]. Also it is necessary to note that according to [5] the first law of thermodynamics for the heat exchange for substances with negative thermal expansion has the following form:

$$\delta Q = dU - PdV. \quad (5)$$

This form differs from that in Eq. 1. In [5] it was shown using experimental data that for substances with negative thermal expansion $C_V > C_p$ and $K_S > K_T$. It means that the traditional result $C_p > C_V$ and $K_T > K_S$ is not that undoubtful. In Table 1 for $T = 260$ K, formally $K_S > K_T$.

STATEMENTS AND DECLARATIONS

Competing Interests

The author has no competing interests to declare that are relevant to the content of this article.

The author has no relevant financial or non-financial interests to disclose. There was no Funding of the manuscript.

Data Availability Statement (DAS)

The data used to generate the results in the paper are available in the paper.

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